below $\frac{18}{12}$; forty-five transverse rows between nape and origin of tail, and thirty-six rows between front of humerus and vent.

The interfrontonasal is transversely diamond-shaped, and has no external plates at its lateral margins. The frontonasals have considerable mutual contact. There are two postnasals; the anterior (and only) canthal descends to the labials, taking the place of the loreal, and there is one large preocular. A postmental follows the symphyseal, and then one pair of infralabials in contact. Two pairs follow, the anterior interrupted by one, the second by two, scales. The auricular opening is nearly as long as the fissure of the eye. The appressed limbs are separated by the space of four ventral cross-rows, or the length of the longest digit of the manus. The tail is of moderate length.

Color of upper surface and sides, brown, the latter a little darker, and bounded above by a narrow black line. A somewhat irregular row of small black spots down the median dorsal line. Below yellowish olive, the scales of the abdomen with black borders, those of the gular and thoracic regions with black centres.

Total length, M. .143; length to auricular meatus, .012; to axilla, .023; to vent, .061.

From the summit of the Pico Blanco (elevation 11,500 feet) in the Eastern Cordillera of Costa Rica; W. M. Gabb.

This species I provisionally identified with the *G. fulvus* of Bocourt, which has been found in Guatemala. The two species are probably nearly allied, but present a difference in the cephalic scutellation, which is of generic value.

Further Illustrations of Central Force.

By Pliny Earle Chase, LL.D.,

Professor of Philosophy in Haverford College.

(*Read before the American Philosophical Society, July 20th*, 1877.)

The establishment of centres of oscillation and harmonic nodes, in an elastic medium, is a necessary consequence of the principle that "a system of bodies in motion must be regarded mechanically as a system of forces or powers which is a perfect representation of all the single powers of which the system is compounded, and this, too, at whatever time or times the component powers may have been introduced into the system." *

But since it is often more difficult to grasp truths which are presented under new aspects, than those which are clothed in familiar garbs, it may be well to glance at some of the most obvious tendencies to nodal action, which result from simple gravitating fall towards a centre. The exami-

* Peirce. Proc. A. A. S., ii, 111.

nation will be the more interesting and suggestive, because like tendencies must exist in all central forces which vary inversely as the square of the distance.

Ennis * has called attention to the fact, that the difference between the velocity of infinite radial fall $(\sqrt{2\,gr})$ and circular-orbital velocity $(\sqrt{gr})$, must be accounted for in some way, and he thinks that it may be sufficient to explain all the phenomena of planetary rotation and revolution. In nebular condensation from r to $\frac{r}{n}$, the increase of radial velocity is $(\sqrt{n}-1)\sqrt{2\,gr}$; the circular-orbital velocity at $\frac{r}{n}=\sqrt{ngr}$; therefore the increase of radial velocity would be sufficient to produce orbital velocity in the periphery of a stationary nebula, when $\sqrt{n}=\sqrt{2}\,(\sqrt{n}-1)$, and $n=\frac{2}{3-2\sqrt{2}}=11.656854$. If r be made to represent, successively, all points between secular aphelion and secular perihelion, in the hypothetical nebulous belts which were condensed into Neptune, Uranus, Saturn and Jupiter, this fall of condensation from Neptune would give orbital velocities in the asteroidal belt; from Uranus, in the Mars belt; from Saturn, in the Venus belt; and from Jupiter, in the Mercury belt. Earth, as I have already shown, is at the centre of the primitive inter-asteroidal belt, which appears to have been then broken up by the action of Uranus, Saturn and Jupiter.

Neptune,	$\div\ n$	$=$ 2.577	Astræa,	$=$	2.577
Uranus, s. p.,†	$\div\ n$	$=$ 1.517	Mars,	$=$	1.524
Saturn, s. p.,	$\div\ n$	$=$ .749	Venus, a.,	$=$	.749
Jupiter, s. a.,	$\div\ n$	$=$ .473	Mercury, s. a.,	$=$	.477

This would leave the orbital velocities of the four outer planets to be accounted for by like condensation from an earlier nebulous condition, of which we have no visible evidence, but if the main hypothesis is correct, we may reasonably look for confirmation of a different kind, within the present limits of the solar system. If we consider the *vis viva* of orbital and radial velocity for unit of mass, the *v. v.* added by radial fall from r to $\frac{r}{m}$ is $(m-1)\,gr$, while the *v. v.* added by equivalent orbital contraction is only $\frac{1}{2}\,(m-1)\,gr$, or one-half of the radial addition. A simple nebular condensation from r to $\frac{r}{2}$ would, therefore, add gr to the *v. v.*, which is equivalent to the *v. v.* of circular-orbital revolution at $\frac{r}{2}$. There is, therefore, a tendency to repeated nebular ruptures at $\frac{r}{2}$, $\frac{r}{4}$, $\frac{r}{8}$ .... $\frac{r}{2^m}$.

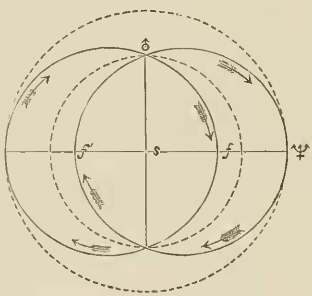
Starting from the present outer limit of our system, Neptune's secular

* "Origin of the Stars."

† a., *aphelion*; p., *perihelion*; s., *secular*.

aphelion (30.46955), these rupturing nodes would occur at 15.23478; 7.61739; 3.80870; 1.90435; .95217; .47608; .23804. The first belt would include Neptune and Uranus; the second, Saturn; the third, Jupiter; the fourth, the asteroids; the fifth, Mars and Earth; the sixth, Venus (grazing also the Earth and Mercury belts); the seventh, Mercury.

After the nebula had assumed a globular form, these rupturing nodes would occasion constant tendencies from opposite extremities of every diameter, to the formation of confocal elliptic orbits, with major axes of $\frac{3r}{2}$ and minor axes of $\sqrt{8}r$. Those ellipses would mutually intersect at $\frac{2r}{3}$, thus tending, through collision of particles, to form a belt at that distance from the centre. The *v. v.* communicated by simple fall from r to $\frac{2r}{3} = \frac{1}{2}gr$, which is equivalent to *v. v.* of circular-orbital revolution at r, and also to the orbital *v. v.* gained by contraction from r to $\frac{r}{2}$. The internal motions and collisions of the particles of the belt would form a condensation of the densest and comparatively inelastic materials, until the whole acquired the mean orbital *v. v.*, $\frac{gr + 2\,gr}{4} = \frac{3\,gr}{4}$, which is the normal orbital *v. v.* at the nodes of aggregating collision, $\frac{2r}{3}$. The following table exhibits the double tendency, to nebular rupture and to nebular aggregation, starting from the point which would account for the orbital velocity of Neptune. The approximation of "B" to the planetary distance which would satisfy Bode's law, and the indications of Neptunian aggregation during direct fall towards the centre, lend new confirmation to the views which I have already expressed, in regard to the *rationale* of Bode's law, and the relative masses of the two outer planets.



	Rupturing Nodes.	Secondary Nodes.	Planets.		
2 × ♆ a*	60.93910	40.62606	"B"	=	38.8
♆ a	30.46955	20.31303	♅ a	=	20.68
$\frac{1}{2}$ ♆ a	15.23478	10.15652	♄ a	=	10.34
$\frac{1}{4}$ ♆ a	7.61739	5.07826	♃ p*	=	4.89

* a secular aphelion; p secular perihelion.

The following tables exhibit the modifying influences of other simple nodes :

$\frac{2}{3}$ ♂	1.0158	⊕	1.0000	$\frac{1}{2}$ ♂	.7618	♀ *a*	.7744
$\frac{2}{3}$ ⊕	.6667	♀ *p*	.6722	$\frac{2}{3} \times \frac{2}{3}$ ♂	.6772	♀ *p*	.6722
$\frac{2}{3}$ ♀	.4822	☿ *a*	.4768	$\frac{1}{2}$ ⊕ *p*	.4661	☿ *a*	.4768
$\frac{2}{3}$ ☿ *a*	.3178	☿ *p*	.2974	$\frac{1}{3}$ ⊕ *p*	.3111	☿ *p*	.2974

In the inter-asteroidal belt and ellipse, bounded by ♂ *a* and ☿ *p:*

Middle of belt,	1.0169	⊕	1.0000
Middle of ellipse,	.7194	♀	.7233

Jupiter is similarly situated in reference to the Neptune-Uranian, and the Uranus-Saturnian ellipses :

Middle of ♆ *a* ♅ *a*	4.8952	Middle of ♅ ♄	4.8224	♃ *p*	4.8863
" ♆ *p* ♅	5.2073	" ♅ ♄ *p*	5.2246	♃	5.2028
" ♆ *a* ♅	5.6430	" ♅ *a* ♄	5.5701	♃ *a*	5.5193

Saturn is similarly situated in reference to the Neptune-Saturnian and Sun-Uranian ellipses :

Mid. ☉ ♅ *p*	8.8440			♄ *p*	8.7345
" ☉ ♅	9.5918	Mid. ♄ *a* ♆ *p*	9.6275	♄	9.5389
" ☉ ♅ *a*	10.3396	" ♄ *p* ♆ *p*	10.4319	♄ *a*	10.3433

There are, doubtless, many other results of early inter-orbital action, especially in connection with collisions in confocal ellipses, which would furnish interesting subjects of investigation. For example, when the Jupiter belt was completely severed (♃ s. p.), and the Earth and Venus belts were beginning to form (s. a.), the orbital collisions were near the limits of the Mars belt.

Elliptic collision	♃ s. p.	⊕ s. a.	1.753	♂ s. a.	1.736
" "	♃ s. p.	♀ s. a.	1.337	♂ s. p.	1.311

If we take the radius of nebular rupturing fall for the surface of Sun's homogeneous luminiferous atmosphere (2 $\times$ light-modulus), and reduce it in the ratio of mean radially-varying to uniform-circular velocity $\left\{\frac{2}{\pi}\right\}$*, rupturing nodes ($\frac{1}{2}$) and falls of condensation ($1 \div 11.656854$) give the following table :

		1st Cond. Fall.	2d Cond. Fall.	Rad. Vec.	
4 M $\div$ π	2807.4	240.84	20.67	♅ *a*	20.68
2 " "	1403.7	120.42	10.33	♄ *a*	10.34
1 " "	701.9	60.21	5.17	♃	5.20
$\frac{1}{3}$ " "	350.9	30.10	....	♆	30.03
$\frac{1}{4}$ " "	175.5	15.05	1.29	♂ *p*	1.31
$\frac{1}{8}$ " "	87.7	7.53	.65	♀ *p*	.67

This seems to point, like the Neptune-Saturnian ellipse in a previous

* If synchronous undulations are interrupted by an obstacle, so as to produce accelerated motion towards a centre, the mean radius of variable motion is $\frac{2}{\pi}$ the radius of corresponding uniform motion.

comparison, and like the present comparatively nebulous condition of Saturn itself, to Saturn as an important centre of early ring aggregation, as if our nebula were, at first, a ring vortex. The indication is confirmed by the similar densities of Saturn and Neptune; the similar densities of Uranus, Jupiter and Sun; the fact that "these four planets form a system by themselves, which is practically independent of the other planets of the system;"* the present approximate accordance between the transit of light through the Uranus-Telluric major-axis and the limit of planetary velocity at Sun's surface; and the following comparison between the 2d and 3d condensation falls:

Rad. Vec.		2d C. Fall.	3d C. Fall.	Rad. Vec.	
♅ a	20.68	20.67	1.77	♂ a	1.74
♄ a	10.34	10.33	.89	⊕ p	.93
♃	5.20	5.17	.44	☿ a	.48
Ast.	2.59	2.59	.22	V a	?

If the 3d fall had been counted from Saturn's secular perihelion instead of from his secular aphelion, the distance would have been .75, Venus's mean aphelion being .75.

The peculiar indication of the Uranus-Telluric ellipse, the central position of Earth in the belt of greatest density, and the absence of any explicit indication of our planet in most of the foregoing comparisons, suggest the possibility that its place may have been fixed by a special law. Its secular perihelion (.93226) is near the fifth rupturing node of Neptune's mean distance ($30.03386 \div 2^5 = .93856$).

The stellar-Solar parabola points to a time when a Centauri may have been at a nebular rupturing point, relatively to the Sun. The lowest and highest estimates for $\frac{2}{\pi}$ a Centauri, are, respectively, 28905200 and 30895100 solar radii. The seventh fall of condensation $(1 \div 11.656854)^7$, would give .9883 and 1.0564, showing a closeness of approximation to the present solar radius which can hardly be thought accidental. As there are two falls of condensation between $\frac{2}{\pi}$ Earth and Sun, there are five falls between a Centauri and Earth; the extreme range of estimates for a Centauri $\div 11.656854^5$ being between .9818 and 1.0494 times Earth's mean radius vector. Both of these points are within the Earth belt ($p = .9323$, $a = 1.0677$).

Neptune's secular eccentricity seems to have been determined by the combined influence of condensation-fall, orbital collision, and rupturing nodes. For Neptune's secular perihelion $\div 11.656854 = 2.53912$; $\frac{2}{3}$ sec. aph. $\div 2^3 = 2.53913$.

The *gegenschein*, and other indications that the Zodiacal light may be partly owing to the remains of an early terrestrial ring, may naturally lead us to look for evidences of residuary activity in some of the outer

* Stockwell; *Smith. Con.* 232, xiii.

planets. A radial oscillation at Uranus's secular aphelion would be accomplished in $10.3396^{\frac{3}{2}} = 33.247y$; a circular revolution at Saturn's secular aphelion, in $10.3433^{\frac{3}{2}} = 33.265y$; a circular revolution, at Jupiter's *mean* perihelion, in $4.9872^{\frac{3}{2}} = 11.108y$. The November meteoric cycle is $33.25y$; the Wolf Sun-spot cycle, $11.07y$.

There is a noteworthy numerical correspondence between the seven rupturing nodes within the planetary belt, and the seven condensation-falls from α Centauri to $\frac{\pi}{2}$ Sun. The fifth node and the fifth fall both come within the Earth belt.

If we suppose seven successive transformations of uniform into variable velocity, before the determination of the present solar mass and light-modulus (M), and five condensation falls ($n = 1 \div 11.656854$) after each transformation, we have the following approximations :

π^7 M $\div$ n^5	30.941	♆ a	30.470
π^6 M $\div$ n^5	9.849	♄	9.548
π^5 M $\div$ n^5	3.135	Hygeia	3.121
π^4 M* $\div$ n^5	.998	⊕	1.000
π^3 M $\div$ n^5	.318	☿ p	.297
π^2 M $\div$ n^5	.101		
π M $\div$ n^5	.032		

The probability of undulating gravitating action is increased by the investigations of Bjerknes, who has shown (Comptes Rendus, lxxxiv, 1377) that two spheres, having concordant pulsations, attract each other inversely as the square of the distance ; and that they repel each other according to the same law if their pulsations are opposed.

The use of the parabola † in representing expanding action is recognized by H. Ste.-Claire Deville, who states, in considering cases where vapor-densities vary with the temperature, that "the movement of a material point, taken in the expanding material, may be accurately enough represented by a parabolic function of the second degree already employed by M. Fizeau." (Comptes Rendus, lxxxiv, 1257). Deville hopes to employ the resulting relations usefully in expounding some principles of Thermo-Chemistry.

The hypothesis that the radial *vis viva* of mean rectilineal velocity may be taken as the representation of increments of heat under constant volume, while the radial *vis viva* of synchronous constant velocity, will represent simultaneous increments of heat under constant pressure, ‡ assumes that the gaseous condition is perfect.

If the Sun were nebulously diffused to 2 ♆, the equal centrifugal and

* π^4 M = distance of α Centauri.

† *Ante*, xvi, 507. ‡ *Ante*, xiv., 651.

centripetal action and reaction would tend to produce a belt of "constant volume," with an inner limit at 1.4232Ψ from the equatorial surface, or .5768Ψ (= .2884 r) from the nucleal centre. The consequent thermodynamic undulations, the *vis viva* of central fall, the vertical collisions at $\frac{2}{3}$ r, and Ennis's centripetal momentum, would all be simultaneously operative, and the present evidence of their past activity is unmistakable. For if we designate the primitive radius ($2\ \Psi_5$) by a; the thermodynamic ratio (.2884) by $\frac{1}{m}$; the *vis viva* ratio by $\frac{1}{2}$; the collision ratio by $\frac{2}{3}$; the Ennis, or momentum ratio ($1 \div 11.656854$) by $\frac{1}{n}$; secular perihelion, mean perihelion, mean, mean aphelion and secular aphelion respectively, by subscript $_{1,2,3,4,5,}$ we find the following primary accordances:

$a = 2\ \Psi_5$	60.939		
$\frac{2}{3}\ a$	40.626		
$\frac{1}{2}\ a$	30.470	$Ψ_5$	30.470
$\frac{1}{m}\ a$	17.575	♅$_1$	17.688
$\frac{1}{n}\ a$	5.228	♃$_3$	5.203

The inner limit of the Neptune-Uranian belt, the controlling centre of planetary mass, and, as we shall presently see, the nebular surfaces which were to determine subsequent planetary aggregations, were thus marked out, within less than one per cent., "in the beginning."

The order of time in which these dissociating influences would be completed, would be $\frac{1}{m}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{n}$. Second and third dissociations present the following agreements:

$\frac{2}{3}\ \frac{1}{m}\ a$	11.717		
$\frac{2}{3}\ \frac{1}{2}\ a$	20.313	♅$_4$	20.044
$\frac{1}{2}\ \frac{1}{m}\ a$	8.788	♄$_1$	8.734
$\frac{1}{m} \cdot \frac{1}{m}\ a$	5.069	♃$_4$	4.978
$\frac{1}{n} \cdot \frac{1}{2}\ a$	2.614	Ast.	....
$\frac{1}{n} \cdot \frac{2}{3} \cdot \frac{1}{2}\ a$	1.743	♂$_5$	1.736
$\frac{1}{n} \cdot \frac{1}{m}\ a$	1.508	♂$_3$	1.524
$\frac{1}{n} \cdot \frac{2}{3}\ \frac{1}{m}\ a$	1.005	⊕$_3$	1.000
$\frac{1}{n} \cdot \frac{1}{2} \cdot \frac{1}{m}\ a$	.754	♀$_4$	.749
$\frac{1}{n} \cdot \frac{1}{n}\ a$	.448	☿$_4$	.455

Second dissociations, therefore, approximately fixed cardinal positions of

♅, ♄, ♃, the asteroidal belt, ♂, and ☿ ; third dissociations, of ♂, ⊕ and ♀.

Numerous other interesting relations, of a similar nature, may be traced at successive stages of nebular condensation, of which some examples are given in the following table :

$\frac{1}{m}$	Ψ_4	8.749	♄$_1$	8.734
$\frac{1}{m}$	♅$_3$	5.533	♃$_5$	5.519
$\frac{1}{m}$	♂$_4$	.474	☿$_5$	.477
$\frac{1}{m}$	⊕$_4$	.298	☿$_1$	.297
$\frac{1}{m^2}$	♅$_2$	1.524	♂$_3$	1.524
$\frac{1}{m^2}$	♄$_1$	.726	♀$_3$	.723
$\frac{1}{m^2}$	♃$_5$	.459	☿$_4$	.455
$\frac{1}{m^3}$	Ψ_3	.720	♀$_3$	.723
$\frac{1}{n}$ .2	Ψ_4	5.205	♃$_3$	5.203
$\frac{1}{n}$	♅$_3$	1.646	♂$_4$	1.644
$\frac{1}{n}$	♄$_1$	.749	♀$_4$	.749
$\frac{1}{n}$	♃$_5$	.473	☿$_5$	.477
$\frac{2}{3}$	⊕$_3$	.667	♀$_1$	.672
$(\frac{2}{3})^2$	♂$_5$	.772	♀$_5$	.774
$(\frac{2}{3})^2$	⊕$_5$	.475	☿$_5$	.477
$(\frac{2}{3})^2$	♀$_1$	.299	☿$_1$	.297
$(\frac{2}{3})^3$	Ψ_5	9.028	♄$_2$	9.078
$(\frac{2}{3})^3$ 2.	Ψ_3	9.558	♄$_3$	9.539
$(\frac{2}{3})^3$	♅$_2$	5.428	♃$_4$	5.427
$(\frac{2}{3})^3$	♃$_5$	1.635	♂$_4$	1.644
$(\frac{2}{3})^3$	♂$_1$	.388	☿$_3$	.387
$(\frac{2}{3})^3$	⊕$_3$	.296	☿$_1$	.297
$(\frac{2}{3})^4$	♄$_1$	1.725	♂$_5$	1.736
$(\frac{2}{3})^4$	♃$_4$	1.075	⊕$_5$	1.068
$(\frac{2}{3})^4$	♃$_1$	.965	⊕$_2$	.966
$(\frac{2}{3})^5$	♄$_4$	1.317	♂$_1$	1.311
$(\frac{2}{3})^5$	♃$_5$	.727	♀$_3$	.723
$(\frac{2}{3})^6$	a	5.197	♃$_3$	5.203
$(\frac{2}{3})^6$	♃$_4$	.476	☿$_5$	.477
$(\frac{2}{3})^7$	♅$_2$	1.072	⊕$_5$	1.068
$(\frac{2}{3})^8$	♅$_3$	.749	♀$_4$	.749
$(\frac{2}{3})^8$	♄$_4$	.390	☿$_3$	.387

$(\frac{2}{3})^9$	♆$_2$	.773	♀$_4$	.774
$(\frac{2}{3})^9$	⛢$_2$	.477	☿$_5$	.477
$(\frac{2}{3})^{10}$	⛢$_2$	.318	☿$_2$	.319
$\frac{1}{2}$	⛢$_5$	10.340	♄$_5$	10.343
$\frac{1}{2}$	⛢$_4$	10.022	♄$_4$	10.000
$\frac{1}{2}$	♄$_4$	5.000	♃$_2$	4.978
$\frac{1}{2}$	♂$_2$	.702	♀$_2$	.698
$\frac{1}{4}$	⛢$_5$	5.170	♃$_3$	5.203
$\frac{1}{4}$	♃$_3$	1.301	♂$_1$	1.311
$\frac{1}{4}$	♂$_3$	.381	☿$_3$	.387
$\frac{1}{8}$	♃$_4$	.678	♀$_1$	.672
$\frac{1}{16}$	♃$_2$	.311	☿$_2$	.319
$\frac{1}{32}$	♆$_1$	.929	⊕$_1$	.932
$\frac{1}{32}$	♄$_3$	.298	☿$_1$	.297
$\frac{1}{64}$	♆$_5$	.476	☿$_5$	.477

The list might be indefinitely extended by admitting a wider range of differences, as well as by various combinations of the four primitive dissociating factors. After rotation was set up, the centre of rotating inertia, to which Alexander first called attention,* asserted its influence, as may be seen by the following comparisons :

$\frac{1}{r}$ †	♆$_4$	19.184	⛢$_3$	19.184
$\frac{1}{r}$	♄$_1$	5.524	♃$_5$	5.519
$\frac{1}{r}$	♂$_4$	1.040	⊕$_1$	1.034
$\frac{1}{r}$	♂$_3$	.964	⊕$_2$	.966
$\frac{1}{r}$	⊕$_5$	.675	♀$_1$	.672
$\frac{1}{r}$	♀$_4$	.474	☿$_5$	.477
$\frac{1}{r}$	♀$_3$	.457	☿$_4$	.455
$\frac{1}{r^2}$	♂$_5$	.695	♀$_2$	.698
$\frac{1}{r^3}$	⛢$_5$	5.231	♃$_3$	5.203
$\frac{1}{r^3}$	⛢$_3$	4.853	♃$_1$	4.886
$\frac{1}{r^3}$	♃$_5$	1.397	♂$_2$	1.403
$\frac{1}{r^3}$	♃$_3$	1.316	♂$_1$	1.311

* *Smithsonian Contributions*, 280.

† $\frac{1}{r} = \sqrt{.1}$

$\frac{1}{\gamma^{3}}$	♂$_{3}$	.385	☿$_{3}$	.387
$\frac{1}{\gamma^{4}}\cdot 2$	♆$_{2}$	9.514	♄$_{3}$	9.539
$\frac{1}{\gamma^{4}}$	♄$_{5}$	1.655	♂$_{4}$	1.644
$\frac{1}{\gamma^{4}}$	♄$_{3}$	1.526	♂$_{3}$	1.524
$\frac{1}{\gamma^{4}}$	♄$_{1}$	1.398	♂$_{2}$	1.403
$\frac{1}{\gamma^{4}}$	♃$_{1}$	.782	♀$_{5}$	.774
$\frac{1}{\gamma^{5}}$	♄$_{3}$	.965	⊕$_{2}$	.966
$\frac{1}{\gamma^{6}}$	⛢$_{5}$	1.323	♂$_{1}$	1.311
$\frac{1}{\gamma^{6}}$	♃$_{2}$	.319	☿$_{2}$	.319
$\frac{1}{\gamma^{7}}$	⛢$_{3}$	.776	♀$_{5}$	.774
$\frac{1}{\gamma^{7}}$	♄$_{3}$	.386	☿$_{3}$	.307
$\frac{1}{\gamma^{8}}$	♆$_{4}$	.777	♀$_{5}$	.774
$\frac{1}{\gamma^{8}}$	⛢$_{1}$	.453	☿$_{4}$	.455
$\frac{1}{\gamma^{9}}\cdot 2$	♆$_{3}$	.973	⊕$_{2}$	.966
$\frac{1}{\gamma^{9}}$	⛢$_{2}$	.297	☿$_{1}$	.297

If we take geometrical, instead of arithmetical means, and place $\frac{1}{2}a$ at Neptune's mean aphelion instead of his secular aphelion, the influence of orbital collisions on positions of intra-asteroidal planets becomes still more striking. For we find, as theoretical (T) and observed (O) values :

	O.		T.	(T–O). ÷ T.
♃$_{3}$	5.203	$\cdot\frac{a}{n}$	5.223	+.004
♂$_{3}$	1.524	$(\frac{2}{3})^{3}\cdot\frac{a}{n}$	1.548	+.016
⊕$_{4}$	1.033	$(\frac{2}{3})^{4}\cdot\frac{a}{n}$	1.032	—.001
♀$_{2}$	.697	$(\frac{2}{3})^{5}\cdot\frac{a}{n}$	.688	—.014
☿$_{4}$	.452	$(\frac{2}{3})^{6}\cdot\frac{a}{n}$	.459	+.015
Mean	1.209		1.213	+.004

Comparing the positions of inter-Uranian planets which are most correctly represented in the foregoing tables, and taking the geometrical means for the five positions of each planet, we find :

	O.	T.	(T-O) ÷ T.
♄	9.521	9.512	—.0009
♃	5.197	5.196	—.0002
♂	1.516	1.517	+.0014
⊕	.999	1.001	+.0021
♀	.722	.721	—.0012
☿	.380	.380	+.0001
Mean	3.567	3.567	+.0002

A similar closeness of accordance is shown by comparing the positions of the intra-Nepturian planets which appear to be most typical :

	O.			T.	(T-O) ÷ T.
♅$_3$	19.184	$\frac{1}{r}$	Ψ$_4$	19.186	+.0001
♄$_5$	10.343	$\frac{1}{2}$	♅$_5$	10.340	—.0004
♃$_4$	5.427	$(\frac{2}{3})^3$	♅$_2$	5.429	+.0003
♂$_3$	1.524	$\frac{1}{m^2}$	♅$_2$	1.524	+.0002
⊕$_2$	.966	$\frac{1}{r^5}$	♄$_3$	.965	—.0009
♀$_4$	.749	$(\frac{2}{3})^8$	♅$_3$	.749	—.0005
☿$_5$	.477	$(\frac{2}{3})^9$	♅$_2$	.477	—.0003
Mean	2.473			2.478	—.0002

The variation of the nucleal radius as the $\frac{3}{4}$ power of the atmospheric radius,* may furnish an explanation of results which seem to have been obtained nearly simultaneously, by Silas W. Holman (A. A. A. S. June 14, 1876 ; P. Mag, Feb., 1877 ; p. 81), and E. Warburg (Pogg. Ann. clix, 415 ; communicated 9th July, 1876). Holman concludes, from the results of a number of careful experiments, that the "viscosity of air increases proportionally to the 0.77 power nearly, of the absolute temperature, between 0° and 100° C." The extreme range of his results is .738 to .799. Warburg, from experiments both with hydrogen and with air, deduces the exponents between 20° and 100°, .76 for air (the extremes being .74 and .76), and "about $\frac{2}{3}$" for hydrogen (the extremes being .57 and .65). The closeness, the narrow range, and the mutual confirmation of these independent results, as well as the new analogy between molar and molecular forces, which seems to be indicated by the atmospheric exponents, are all interesting. The viscous particles, so far as they are affected by the same movements, may be compared to the rotating particles of a solid nucleus ; the thermal undulations, in a supposed æthereal medium, present a like analogy to the motions of an elastic atmosphere. The well known anomalies in the elasticity of hydrogen are in accordance with its viscosity. War-

* *Ante*, xiv, 305 *et. al.*

burg's extremes (hydrogen .57, air .76) seem to point towards secondary nucleal and atmospheric relations between air and hydrogen.

In my identification of the velocity of solar dissociation with the velocity of light,* although the conception of successive wave impulses seems most natural, it is by no means essential. If the pressure of the ultimate force is constant, the result is the same. The ratio of the velocity of dissociation to the velocity of perfect fluidity,† is approximately illustrated by Draper's estimate of the ratio between the temperature of glow (977° F., or 1436° from absolute 0°) and the temperature of fluidity (32° F., or 491° from absolute 0°; $1436 \div 491 = 2.9$). Here complete fluidity is compared with incipient glow. The ratio $\pi : 1$ would require an additional allowance of 107°, or about 7.5 per cent., for the difference between the temperature of complete and incipient glow. If the comparison were made at 0° F., we should have $1436 \div 459 = 3.13$.

The *vis viva* of terrestrial dissociation being equivalent to ½ the *v. v.* of incipient planetary dissociation at the Sun,‡ the temperature ratio of water vaporization to dissociation furnishes another illustration of a similar character. Deville (C. Rendus, lxxxiv, 1259) quotes the estimates made by himself and Debray (2500°), and by Bunsen (2800°), of the temperature at which nearly half of the vapor of water is reduced to its elements, hydrogen and oxygen. The ratio 2800° : 100° is a very probable estimate of the ratio between solar and terrestrial superficial gravitation.

NOTE.—August 23, 1877. In consequence of a remark near the opening of the foregoing paper, Dr. Draper recently proposed that I should test some of my views by an examination of the solar spectrum. I accordingly undertook a preliminary investigation, which has already yielded the following results:

In the harmonic progression, $\frac{c}{n}$, $\frac{c}{n+a}$, $\frac{c}{n+2a}$, etc., let c = wavelength of Fraunhofer line A = 761.20 millionths of a millimetre; n = 1.0153; a = .0918; and we find the following accordances:

Numerator.	Denominators.	Quotients.	Observed values.
761.20	$n + a$	687.75	687.49 = B
	$n + 3a$	589.89	589.74 = D^1
	$n + 6a$	486.14	486.52 = F
	$n + 10a$	393.79	393.59 = H^1

The "observed values" are the wave-lengths, as determined by Dr. Wolcott Gibbs (Amer. Jour. Sci. [2] xliii, 4). The lines between A and B have not been studied sufficiently to fix their wave-lengths; it seems likely that $A \div n$ may be a *bright* line, and thus belong to the field of investigation which Professor Draper has so brilliantly opened. The greatest difference between the above theoretical and observed values, is

* *Ante*, xvi, 307, *et al.* † Ib. p. 305. ‡ Ib. p. 307–8.

less than four ten-millionths of a millimetre, and, therefore, very far within the limit of probable errors of observation.

My papers on planetary harmonies have shown that alternate planetary positions manifest the greatest simplicity of law, intermediate positions being modified by requirements of mutual equilibrium, which help to give stability to the system. The same thing seems to be true of the Fraunhofer lines. The "figurate" symmetry of the above divisor differences ($1\,a$, $3\,a$, $6\,a$, $10\,a$) is especially noticeable, and suggestive of my equation between the principal planetary masses :

$(\text{Neptune})^{1} \times (\text{Uranus})^{3} \times (\text{Jupiter})^{6} \times (\text{Saturn})^{-10} = 1.$

After finding this relation among the most important lines, I sought for traces of the "morning-star" music among the subordinate lines, with the following result: I have introduced Kirchhoff's scale-measurements, in order that the lines may be identified without the necessity of reference to Dr. Gibbs's papers.

Divisors.	Quotients.	Observed values.	Scale measurem'ts.
$n + 2\,a$	635.07	634.05	783.8
$n + 4\,a$	550.72	550.70	1306.7
$n + 5\,a$	516.42	517.15	1655.6
$n + 7\,a$	459.22	458.66	2436.5
$n + 8\,a$	435.12	435.67	2775.7
$n + 9\,a$	413.43	(413.76)	(?)

There is no single line corresponding to the harmonic denominator $n + 9\,a$. The bracketed number is the arithmetical mean between Kirchhoff line $2869.7 = 430.37$, and $\text{H} = 397.16$. This again, may either indicate a *bright* line, or it may await future discovery for a true interpretation.

The equality, which I had previously pointed out, between the average limiting velocities of solar centrifugal and tangential dissociation, and the velocity of light, induced me to apply the same harmonic series to the solar system. In some of the papers on cosmical and molecular force, which I have had the honor of communicating to the society (Proc. Soc. Phil. Amer. vol. xiii.), I had taken steps in this direction, but they were comparatively feeble, for want of sufficient definite guidance. They had, however, shown very clearly, that, in ultimate physical generalizations, the study of elastic reaction is quite as needful as the study of centripetal action, and *vice versa*. One of the most important facts, in connection with such comparative study, is the variation of elastic density in geometrical ratio, when distance varies in arithmetical ratio. In making an operative application of the spectral harmonic series, the several terms should therefore be taken exponentially, and the greatest activity should be looked for at inter-nodes, and presumably nearly midway between successive nodes. The Sun's radius was naturally suggested as a fundamental unit.

The process of calculation is nearly as simple as Columbus's egg, but, on account of its novel application, it may be well to give it in full. The common astronomical unit is Earth's mean radius vector; its value, in units of solar radius, is 214.86. The harmonic exponential numerator, is Neptune's mean radius vector, which is 30.03386 astronomical units, or 30.03386 × 214.86 = 6453.06 solar radii. The logarithm of 6453.06 is 3.809766; log. log. 6453.06 = log. 3.809766 = .580897. By the same method we find log. log. Uranus = .558210; .580897 — .558210 = .022687 = log. 1.0536. Uranus's mean radius vector represents, therefore, the 1.0536th root of Neptune's mean radius vector, and 1.0536 is the denominator of the first planetary fractional exponent. The first mid-nodal denominator, in the foregoing spectral-line series, between A ÷ 1 and A ÷ $(n + a)$ is $(1 + 1.1068) \div 2 = 1.0534$; the second mid-nodal denominator is $(n + a + n + 2a) \div 2 = 1.1527$; and so on, until we reach the sixth denominator, when, perhaps on account of great nebular condensation, the harmonic denominator-differences become $\frac{5}{4}$ of .0918, instead of .0918, bringing a second exact correspondence between the spectral and planetary denominators in the orbit of Venus. The following table contains all the figures that are required for the whole calculation:

Expon'l Den'rs.	Log. Den'rs.	Log. log. *r. vec.*	Log. *r.* I.	Log. *r.* II.	Theoretical.	Observed.
1.0534	.022593	.558304	3.61689	1.28473	19.263	19.184
1.1527	.061716	.519181	3.30576	.97360	9.410	9.539
1.2445	.094994	.485903	3.06128	.72912	5.359	5.427
1.3368	.126066	.454831	2.84991	.51775	3.294	?
1.4281	.154758	.426139	2.66771	.33555	2.165	?
1.5199	.181815	.399082	2.50658	.17442	1.494	1.524
1.6346	.213412	.367485	2.33070	$\bar{1}$.99854	.997	1.000
1.7494	.242889	.338008	2.17775	$\bar{1}$.84559	.701	.698
1.8641	.270469	.310428	2.04375	$\bar{1}$.71159	.515	.510
1.9789	.296424	.284473	1.92519	$\bar{1}$.59303	.392	.387

The log. logs., in the third column, are obtained by subtracting the logs. of the exponential denominators (column 2) from the log. log. of the exponential numerator (.580897). Column 4 contains the antilogs. of column 3; column 5 is column 4 reduced to logs. of Earth's mean radius-vector, by substracting log. 214.86 = 2.332155; column 6 contains the antilogs. of column 5. Column 7 gives the mean distances of Uranus, Saturn, Mars, Earth, and Mercury; the mean aphelion of Jupiter; the mean perihelion of Venus; and the arithmetical mean between Mercury's secular perihelion, and Venus's mean distance.

We are now prepared to find the significance of the remaining Fraunhofer lines, which is shown in the following table:

Line.	Wave Length.	Denominator.	Planetary Den'rs.	Theoretical Den'rs.
C	656.67	1.1590	1.1576 = Sat. p.*	
E	527.38	1.4434	Asteroidal.	
b	517.70	1.4704		$1.4740 = n + 5a$
G	431.03	1.7660	1.7640 = Ven. s.p.	
H	397.16	1.9166	1.9139 = Mer. a.	

* p., *mean perihelion;* s. p., *secular perihelion;* a., *mean aphelion.*

The following table gives a comparative view of the spectral and planetary series:

Spectral	Differences.	Planetary	Diff.
α 1.0000	$\frac{1}{6}\,a$	1.0000	$\frac{7}{12}\,a$
β 1.0150	a	$\frac{1}{2}\,(\alpha + \gamma)$ 1.0534	$\frac{13}{12}\,a$
γ 1.1068	a	$\frac{1}{2}\,(\gamma + \delta)$ 1.1527	a
δ 1.1986	a	$\frac{1}{2}\,(\delta + \varepsilon)$ 1.2445	a
ε 1.2904	a	$\frac{1}{2}\,(\varepsilon + \zeta)$ 1.3363	a
ζ 1.3822	a	$\frac{1}{2}\,(\zeta + \eta)$ 1.4281	a
η 1.4740	a	$\frac{1}{2}\,(\eta + \theta)$ 1.5199	$\frac{5}{4}\,a$
θ 1.5658	a	$\frac{1}{2}\,(\theta + \iota) + \frac{1}{4}\,a$ 1.6347	$\frac{5}{4}\,a$
ι 1.6576	a		
κ 1.7494	a	κ 1.7494	$\frac{5}{4}\,a$
λ 1.8412	a		
μ 1.9330	a	$\frac{1}{2}\,(\lambda + \mu) - \frac{1}{4}\,a$ 1.8641	$\frac{5}{4}\,a$
ν 2.0248	a	$\frac{1}{2}\,(\mu + \nu)$ 1.9789	$\frac{5}{4}\,a$
o 2.1166	a	$\frac{1}{2}\,(\nu + o) + \frac{1}{4}\,a$ 2.0936	$\frac{5}{4}\,a$
π 2.2084		π 2.2084	

In the fundamental harmonic denominators, it will be seen that $a = 6\,n$, and 6 is the figurate exponent of Jupiter in the equation of planetary masses. The value of n is the quotient of (Jupiter $\times$ perihelion radius-vector) by (Sun $\times$ solar radius). The significance of this quotient is obvious, on account of the preponderating influence of the two controlling members of our system. It becomes still more interesting upon examining the portion of the spectrum which represents Jupiter's most powerful reaction against solar action.

As the harmonic basis is Jupiter's *present* perihelion, it seems likely that there may be some changes in the relative positions of the spectral lines, with Jupiter's changing eccentricity. As this change is less than $\frac{1}{375}$ of one per cent. per annum, its influence cannot be detected by direct observation. But it may be worth while to institute careful comparisons between solar spectra taken at our perihelion, aphelion, perijove and apojove, in order to find whether the lines are modified in any way by Earth's position relatively to Sun and Jupiter.